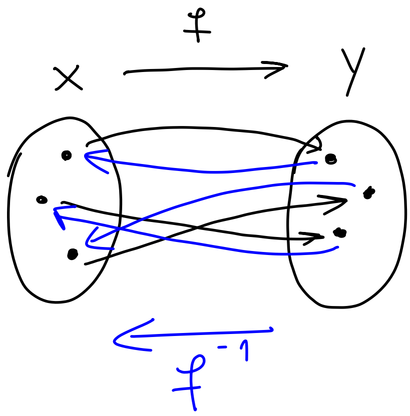
Função inversa

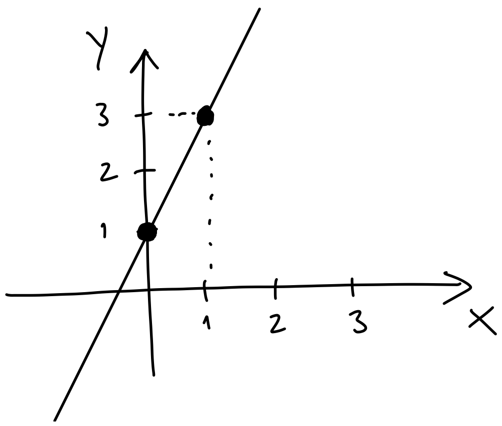


(todo)
cada elemento de X está associado a um único elemento de Y.

$$f^{-1}(f(x)) = x \quad , \quad f(f^{-1}(y)) = y$$

$$\sqrt{(x^2)} = x \qquad (\sqrt{y})^2 = y$$

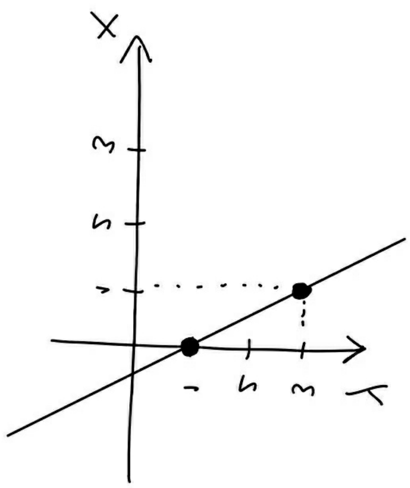
Exemplo: 1) $f(x) = 2x+1$



$$f(0) = 2 \cdot 0 + 1 = 1 \Rightarrow (0,1) \in G(f)$$

$$f(1) = 2 \cdot 1 + 1 = 3 \Rightarrow (1,3) \in G(f)$$

$$f^{-1}(y) = ?$$

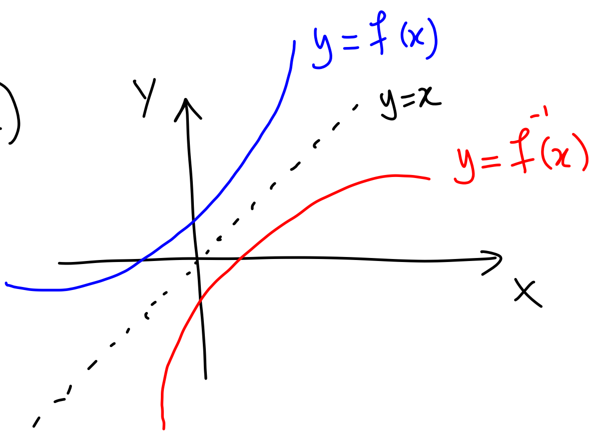


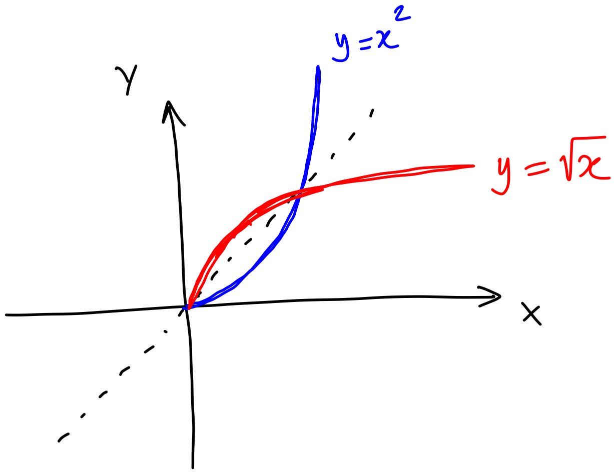
$$y = 2x+1 \iff y - 1 = 2x$$

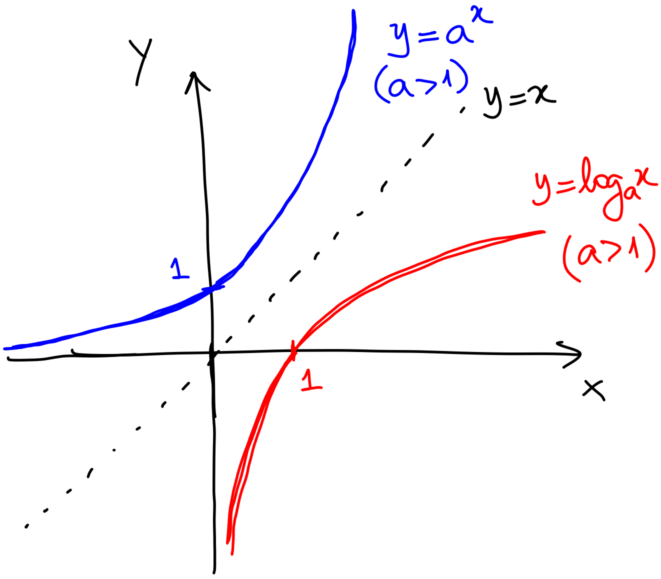
$$\iff x = \frac{y-1}{2} \iff x = \frac{1}{2}y - \frac{1}{2}$$

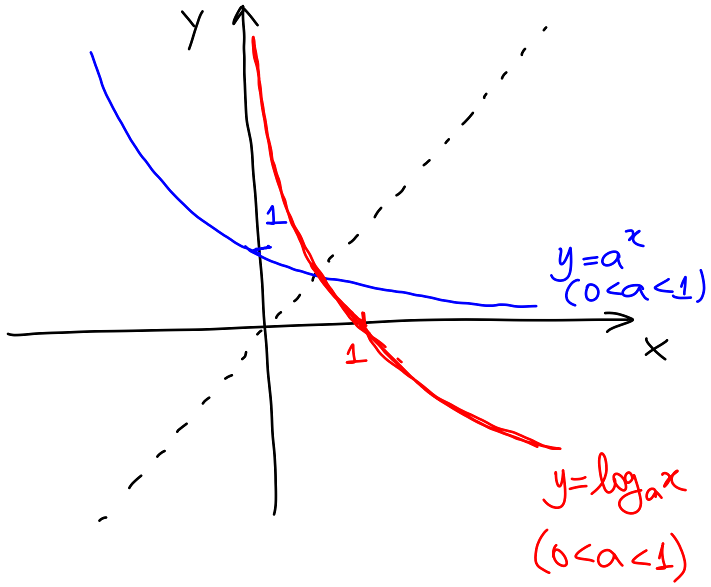
$$\therefore f^{-1}(y) = \frac{1}{2}y - \frac{1}{2}$$

2)









Exercício: A expressão $M(t) = 200 \cdot e^{-\frac{t \ln 2}{30}}$ dá a massa em gramas de césio 137 que restará de uma quantidade inicial após t anos de decaimento radioativo.

a) Quantos gramas havia inicialmente?

b) Quantos gramas permanecem depois de 10 anos?
$\left(\frac{1}{\sqrt[3]{2}} \approx 0{,}794\right)$

c) Quantos anos levará para reduzir pela metade a quantidade inicial de césio 137?

a) $M(0) = 200 \cdot e^{-\frac{0 \cdot \ln 2}{30}} = 200\,g$

b) $M(10) = 200 \cdot e^{-\frac{1\cancel{0} \cdot \ln 2}{3\cancel{0}}} = 200 \cdot e^{-\frac{1}{3}\ln 2} = 200 \cdot e^{\ln(2^{-1/3})}$

$= 200 \cdot 2^{-1/3} = 200 \cdot \frac{1}{2^{1/3}} = 200 \cdot \frac{1}{\sqrt[3]{2}} \approx 200 \cdot 0{,}794 = 158{,}8g$

c) $M(t) = \frac{M(0)}{2} = \frac{200}{2} = 100 \Rightarrow 200 \cdot e^{-\frac{t \ln 2}{30}} = 100$

$$\Rightarrow e^{-\frac{t\ln 2}{30}} = \frac{1\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}} = \frac{1}{2} \Rightarrow \ln\left(e^{-\frac{t\ln 2}{30}}\right) = \ln\left(\frac{1}{2}\right)$$

$$\Rightarrow -\frac{t \cdot \ln 2}{30} = \ln\left(\frac{1}{2}\right) = \cancel{\ln 1} - \ln 2 = -\ln 2$$

$$\Rightarrow t \cdot \ln 2 = 30 \cdot \ln 2 \Rightarrow t = 30 \text{ anos}$$

Exercício 4. Determine os valores de x para os quais exista:

a) $\log_x(x-1)$.

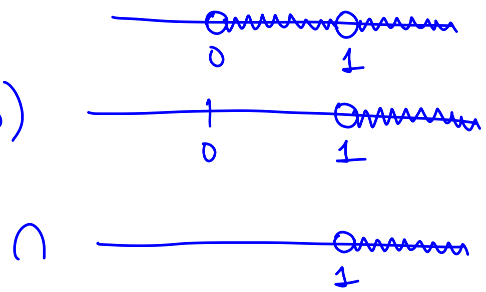
b) $\log_{(x^2-4)} 3$.

$\log_a x \Rightarrow x > 0$

a) $x > 0$ e $x \neq 1$ (base)

e $x - 1 > 0 \Rightarrow x > 1$ (argumento)

$\therefore \underline{\underline{x > 1}}$



b) $x^2 - 4 > 0$ e $x^2 - 4 \neq 1$

$x^2 > 4$ $\qquad$ $x^2 \neq 5$

$\sqrt{x^2} > \sqrt{4}$ $\qquad$ $\sqrt{x^2} \neq \sqrt{5}$

$|x| > 2$ $\qquad$ $|x| \neq 5$

$x > 2$ ou $x < -2$ $\qquad$ $x \neq 5$ e $x \neq -5$